

On New class of Contra Continuity in Soft Micro Topological Spaces

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Abstract: In this paper, we introduce the concept of Contra Soft Micro Continuity in Soft Micro Topological Spaces. Furthermore, we also define Soft Micro Irresolute and Soft Contra Irresolute Soft Micro Functions. The fundamental properties of these concepts are investigated and discussed.

Keywords: Soft Micro Irresolute Functions, Contra Soft Micro Continuity, Contra Soft Micro Irresolute, Contra Soft Micro Semi Continuity, Contra Soft Micro Pre Continuity, Contra soft Micro β – Continuity.

1. INTRODUCTION

The concept of soft set theory introduced by Molodtsov [1] in 1999 to deal with uncertainty and emerged as a mathematical framework designed to handle imprecise, indeterminate data. Maji et al [2] proposed several operations on soft sets. Muhammad Shabir and Munnaza naz [4] introduced the concept of soft topological spaces in 2011. In 2012, Lellis Thivagar and Carmel Richard [5] introduced nano topology an extension of set theory that aids in analyzing intelligent systems with incomplete or insufficient information. In 2017, S.S. Benchalli, P.G Patil, Niveditha, N.S. Kabbur and J. Pradeepkumar [6-8] introduced soft nano topological spaces. In 2019, Chandrasekar [9] introduced the concept of micro topology. [3] S. Deepika and J. Arul Jest introduced the concepts Soft Micro Topological spaces and discuss some properties. [12] S. Deepika and J. Arul Jest introduced Soft Micro Continuity in Soft Micro Topological Spaces. In this paper, we introduce the concept of Contra Soft Micro Continuity in Soft Micro Topological Spaces. Furthermore, we also define Soft Micro Irresolute and Soft Contra Irresolute Soft Micro Functions. The fundamental properties of these concepts are investigated and discussed.

2. PRELIMINARIES

Definition 2.1: [3] Let $(U, \tau_{S_tN}(X), E)$ be a Soft Nano Topological Spaces. Here, $\tau_{S_tM}(X) = \{P \cup (\tilde{P} \cap \delta) / P, \tilde{P} \in \tau_{S_tN}(X) \text{ and } \delta \notin \tau_{S_tN}(X)\}$ is called a Soft Micro Topology on U with respect to X . The triplet $(U, \tau_{S_tM}(X), E)$ is called Soft Micro Topological Space.

Definition 2.2: [3] The soft micro topology $\tau_{S_tM}(X)$ satisfies the following axioms

1. $(U, E), \phi \in \tau_{S_tM}(X)$
2. The Union of the elements of any sub-collection of $\tau_{S_tM}(X)$ is in $\tau_{S_tM}(X)$.
3. The intersection of the elements of any finite sub-collection of $\tau_{S_tM}(X)$ is in $\tau_{S_tM}(X)$.

Then $\tau_{S_tM}(X)$ is called Soft Micro Topology on U with respect to X . The $(U, \tau_{S_tM}(X), E)$ is called Soft Micro Topological Spaces and the elements of $\tau_{S_tM}(X)$ are called Soft Micro open (briefly, S_tMO) set and the complement of a Soft Micro open sets is called a Soft Micro closed (briefly, S_tMC) sets.

Definition 2.3: [3] If $(U, \tau_{S_tM}(X), E)$ is a soft micro topological space with respect to soft subset of (U, E) , then the **Soft Micro interior** of (A, E) is defined as the union of all Soft Micro open subsets of (A, E) and it is denoted by $S_tMint(A, E)$. That is, $S_tMint(A, E)$ is the largest Soft Micro open subset of (A, E) . The **Soft Micro closure** of (A, E) is defined as the intersection of all Soft Micro closed set containing (A, E) and it is denoted by $S_tMcl(A, E)$. That is, $S_tMcl(A, E)$ is the smallest Soft Micro closed set containing (A, E) .

Definition 2.4: [3] Let $(U, \tau_{S_tM}(X), E)$ be a Soft Micro Topological Space and $(A, E) \subseteq (U, E)$. Then (A, E) is said to be

- (i) **Soft Micro Pre Open Set** if $(A, E) \subseteq S_tMint(S_tMcl(A, E))$ and **Soft Micro Pre closed set** if $S_tMcl(S_tMint(A, E)) \subseteq (A, E)$.
- (ii) **Soft Micro Semi Open Set** if $(A, E) \subseteq S_tMcl(S_tMint(A, E))$ and **Soft Micro Semi closed set** if $S_tMint(S_tMcl(A, E)) \subseteq (A, E)$.
- (iii) **Soft Micro β –Open Set** if $(A, E) \subseteq S_tMcl(S_tMint(S_tMcl(A, E)))$ and **Soft Micro β –closed set** if $S_tMint(S_tMcl(S_tMint(A, E))) \subseteq (A, E)$.

Definition 2.5: [12] Let $(U, \tau_{S_tM}(X), E)$ and $(V, \sigma_{S_tM}(Y), F)$ be two soft micro topological spaces. A function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is called **Soft Micro-Continuous** function if $f^{-1}(G, B)$ is soft micro-open in $(U, \tau_{S_tM}(X), E)$ for every soft micro-open set (G, B) in $(V, \sigma_{S_tM}(Y), F)$.

3. SOFT MICRO-IRRESOLUTE FUNCTIONS

Definition 3.1: Consider $(U, \tau_{S_tM}(X), E)$ and $(V, \sigma_{S_tM}(Y), F)$ be a Soft Micro Topological Spaces. A function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is said to be S_tMS –irresolute, if $f^{-1}(G, F)$ is $S_tMSO(U, \tau_{S_tM}(X), E)$ for every (G, F) of $S_tMSO(V, \sigma_{S_tM}(Y), F)$.

Example 3.2: Let $U = \{i, j, k\}, E = \{m_1, m_2, m_3\}$ and $X = \{i\} \subseteq_{S_tM} U$ with $U/R = \{\{j\}, \{i\}, \{k\}\}$. Let $\tau_{S_tN}(X) = \{\phi, (U, E), (K_2, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_4, E) \notin \tau_{S_tN}(X)$. $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_2, E), (K_4, E), (K_6, E)\}$. Let $V = \{m, n, o\}, F = \{n_1, n_2, n_3\}$ and $Y = \{o\} \subseteq_{S_tM} V$ with $V/R = \{\{m, n\}, \{o\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F), (G_4, F)\}$ is a S_tN - Topology on V . Then $\delta = (G_3, F) \notin \sigma_{S_tN}(Y)$. $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_3, F), (G_4, F), (G_7, F)\}$. Define $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be a function defined as $f(i) = n, f(j) = m, f(k) = o$ and $\check{p}: E \rightarrow F$ by $\check{p}(m_1) = n_1, \check{p}(m_2) = n_2, \check{p}(m_3) = n_3$. Then $f^{-1}(G_4, F) = (K_4, E)$ and $f^{-1}(G_5, F) = (K_5, E), f^{-1}(G_3, F) = (K_2, E), f^{-1}(G_6, F) = (K_7, E)$ and $f^{-1}(G_7, F) = (K_6, E)$. Therefore, f is S_tMS –irresolute.

Definition 3.3: Consider $(U, \tau_{S_tM}(X), E)$ and $(V, \sigma_{S_tM}(Y), F)$ be a Soft Micro Topological Spaces. A function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is said to be

- (i) S_tMP –irresolute, if $f^{-1}(G, F)$ is $S_tMPO(U, \tau_{S_tM}(X), E)$ for every (G, F) of $S_tMPO(V, \sigma_{S_tM}(Y), F)$.
- (ii) $S_tM\beta$ –irresolute, if $f^{-1}(G, F)$ is $S_tM\beta O(U, \tau_{S_tM}(X), E)$ for every (G, F) of $S_tM\beta O(V, \sigma_{S_tM}(Y), F)$.

Theorem 3.4: Let $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be a soft micro function. Then the following conditions are equivalent.

- (i) f is soft micro semi-irresolute
- (ii) The inverse image of every soft micro semi closed set $(V, \sigma_{S_tM}(Y), F)$ is soft micro semi-closed in $(U, \tau_{S_tM}(X), E)$.

Proof: The proof is obvious.

Theorem 3.5: Let $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ and $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ be two soft micro function. Then

- (i) $g \circ f: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is S_tMS –irresolute, if f and g is S_tMS –irresolute functions.
- (ii) $g \circ f: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is S_tMP –irresolute, if f and g is S_tMP –irresolute functions.
- (iii) $g \circ f: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is $S_tM\beta$ –irresolute, if f and g is $S_tM\beta$ –irresolute functions.

Proof: Let (N, L) be S_tMS –open set in $(W, \lambda_{S_tM}(Z), L)$. Since g is S_tMS –irresolute, $g^{-1}(N, L)$ is S_tMS - open in $(V, \sigma_{S_tM}(Y), F)$. Since f is S_tMS –irresolute, $f^{-1}(g^{-1}(N, L)) = (gof)^{-1}(N, L)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Therefore, gof is S_tMS –irresolute.

(ii)and (iii) This proof is similar to (i).

Theorem 3.6: Let $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ and $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ be two soft micro function. Then

(i) $gof: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is S_tMS –Continuous, if f is S_tMS –irresolute and g is S_tMS –Continuous.

(ii) $gof: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is S_tMP –Continuous, if f is S_tMP –irresolute and g is S_tMP –Continuous.

(iii) $gof: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is $S_tM\beta$ –Continuous, if f is $S_tM\beta$ –irresolute and g is $S_tM\beta$ –Continuous.

Proof: Let (H, L) be S_tM –open set in $(W, \lambda_{S_tM}(Z), L)$. Since g is S_tMS –Continuous, $g^{-1}(H, L)$ is S_tMS - open in $(V, \sigma_{S_tM}(Y), F)$. Since f is S_tMS –irresolute, $f^{-1}(g^{-1}(H, L)) = (gof)^{-1}(H, L)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Therefore, gof is S_tMS –Continuous.

(ii)and(iii) This proof is similar to (i).

4. CONTRA SOFT MICRO CONTINUOUS FUNCTIONS

Definition 4.1: Consider $(U, \tau_{S_tM}(X), E)$ and $(V, \sigma_{S_tM}(Y), F)$ be a Soft Micro Topological Spaces. A function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is contra soft micro continuous, if the inverse image of every soft micro-open set in $(V, \sigma_{S_tM}(Y), F)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$. That is, if $f^{-1}(G, F)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$ for every soft micro-open set (G, F) of $(V, \sigma_{S_tM}(Y), F)$.

Equivalently, a function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is CS_tM –Continuous iff $f^{-1}(G, F) \in S_tMO(U, \tau_{S_tM}(X), E)$ for every $(G, F) \in S_tMC(V, \sigma_{S_tM}(Y), F)$.

Example 4.2: Let $U = \{i, j, k\}, E = \{m_1, m_2, m_3\}$ and $X = \{i, j\} \subseteq_{S_tM} U$ with $U/R = \{\{j\}, \{i, k\}\}$. Let $\tau_{S_tN}(X) = \{\phi, (U, E), (K_3, E), (K_6, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_2, E) \notin \tau_{S_tN}(X)$. $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_2, E), (K_3, E), (K_5, E), (K_6, E)\}$ and soft micro closed sets are $\phi, (U, E), (K_3, E), (K_4, E), (K_6, E)$ and (K_7, E) . Let $V = \{m, n, o\}, F = \{n_1, n_2, n_3\}$ and $Y = \{m\} \subseteq_{S_tM} V$ with $V/R = \{\{m\}, \{n\}, \{o\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F), (G_2, F)\}$ is a S_tN - Topology on V . Then $\delta = (G_5, F) \notin \sigma_{S_tN}(Y)$. $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_2, F), (G_5, F)\}$. Define $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be a function defined as $f(i) = n, f(j) = o, f(k) = m$ and $\check{p}: E \rightarrow F$ by $\check{p}(m_1) = n_1, \check{p}(m_2) = n_2, \check{p}(m_3) = n_3$. Then $f^{-1}(G_2, F) = (K_4, E)$ and $f^{-1}(G_5, F) = (K_6, E)$ are soft micro closed in $(U, \tau_{S_tM}(X), E)$. That is, the inverse image of every soft micro-open set in $(V, \sigma_{S_tM}(Y), F)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$. Therefore, f is contra soft micro continuous.

Theorem 4.3: Define $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be a soft micro function. Then the following conditions are equivalent.

(iii) f is contra soft micro continuous

(iv) The inverse image of every soft micro closed set in $(V, \sigma_{S_tM}(Y), F)$ is soft micro-open in $(U, \tau_{S_tM}(X), E)$.

Proof: (i) $\Rightarrow$ (ii) Let f be contra soft micro continuous. Let (G, F) be a soft micro closed set in $(V, \sigma_{S_tM}(Y), F)$ and therefore $(G, F)^c$ is soft micro-open in $(V, \sigma_{S_tM}(Y), F)$. By (i) $f^{-1}((G, F)^c)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$. But, $f^{-1}((G, F)^c) = (f^{-1}(G, F))^c$. Hence $f^{-1}(G, F)$ is soft micro-open in $(U, \tau_{S_tM}(X), E)$.

(ii) $\Rightarrow$ (iii) Let (J, F) be a soft micro-open set in $(V, \sigma_{S_tM}(Y), F)$. Then $(J, F)^c$ is soft micro closed in $(V, \sigma_{S_tM}(Y), F)$. By (ii) $f^{-1}((J, F)^c)$ is soft micro-open in $(U, \tau_{S_tM}(X), E)$. Hence $f^{-1}(J, F)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$. Hence f is contra soft micro continuous.

Remark 4.4: The concept of soft micro continuity and contra soft micro continuity are independent as shown in the following example:

Example 4.5: Let $U = \{i, j, k, l\}$, $E = \{m_1, m_2, m_3\}$ and $X = \{i, k\} \subseteq_{S_tM} U$ with $U/R = \{\{j\}, \{i, l\}, \{k\}\}$. Let $\tau_{S_tN}(X) = \{\phi, (U, E), (K_4, E), (K_8, E), (K_{14}, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_{13}, E) \notin \tau_{S_tN}(X)$. $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_4, E), (K_8, E), (K_{13}, E), (K_{14}, E)\}$ and soft micro closed sets are $\phi, (U, E), (K_3, E), (K_4, E), (K_9, E)$ and (K_{13}, E) . Let $V = \{m, n, o, p\}$, $F = \{n_1, n_2, n_3\}$ and $Y = \{n, o\} \subseteq_{S_tM} V$ with $V/R = \{\{m, p\}, \{n\}, \{o\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F), (G_9, F)\}$ is a S_tN - Topology on V . Then $\delta = (G_{15}, F) \notin \sigma_{S_tN}(Y)$. Hence $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_9, F), (G_{15}, F)\}$. Define a function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ by $f(i) = n, f(j) = m, f(k) = p, f(l) = o$ and $\check{p}: E \rightarrow F$ by $\check{p}(m_1) = n_1, \check{p}(m_2) = n_2, \check{p}(m_3) = n_3$. Then $f^{-1}(G_9, F) = (K_8, E)$ and $f^{-1}(G_{15}, F) = (K_{14}, E)$. Hence f is a soft micro continuous function but not contra soft micro continuous. Because, $f^{-1}(G_9, F) = (K_8, E)$ is not soft micro closed in $(U, \tau_{S_tM}(X), E)$, where (G_9, F) is soft micro- open in $(V, \sigma_{S_tM}(Y), F)$. Therefore, f is soft micro continuous but not contra soft micro continuous.

Example 4.6: Let $U = \{i, j, k\}$, $E = \{m_1, m_2, m_3\}$ and $X = \{i, j, k\} \subseteq_{S_tM} U$ with $U/R = \{\{i\}, \{j, k\}\}$. Let $\tau_{S_tN}(X) = \{\phi, (U, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_2, E) \notin \tau_{S_tN}(X)$. $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_2, E)\}$ and soft micro closed sets are $\phi, (U, E)$ and (K_7, E) . Let $V = \{m, n, o\}$, $F = \{n_1, n_2, n_3\}$ and $Y = \{m, n, o\} \subseteq_{S_tM} V$ with $V/R = \{\{m\}, \{n, o\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F)\}$ is a S_tN - Topology on V . Then $\delta = (G_6, F) \notin \sigma_{S_tN}(Y)$. $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_6, F)\}$. Define $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be a function defined as $f(i) = n, f(j) = m, f(k) = o$ and $\check{p}: E \rightarrow F$ by $\check{p}(m_1) = n_1, \check{p}(m_2) = n_2, \check{p}(m_3) = n_3$. Then $f^{-1}(G_6, F) = (K_7, E)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$ and not soft micro-open set in $(U, \tau_{S_tM}(X), E)$. Hence f is contra soft micro continuous function but not soft micro continuous function.

Theorem 4.7: Consider $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ and $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ be the functions then gof is contra soft micro contra continuous if g is soft micro continuous and f is contra soft micro continuous.

Proof: Let (H, L) be a soft micro-open set in $(W, \lambda_{S_tM}(Z), L)$. Since g is soft micro continuous, $g^{-1}(H, L)$ is soft micro-open in $(V, \sigma_{S_tM}(Y), F)$. Since f is contra soft micro continuous, $f^{-1}(g^{-1}(H, L))$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$. That is $(gof)^{-1}(H, L)$ is soft micro closed in $(U, \tau_{S_tM}(X), E)$. Therefore, gof is contra soft micro contra continuous.

Remark 4.8: The composition of two contra soft micro continuous functions need not be contra soft micro continuous as the following example shows.

Example 4.9: Let $U = \{i, j, k, l\}$, $E = \{m_1, m_2, m_3\}$ and $X = \{i, j, k\} \subseteq_{S_tM} U$ with $U/R = \{\{l\}, \{i, j\}, \{k\}\}$. Let $\tau_{S_tN}(X) = \{\phi, (U, E), (K_{12}, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_8, E) \notin \tau_{S_tN}(X)$. $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_2, E), (K_8, E), (K_{12}, E)\}$ and $S_tMC(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_5, E), (K_9, E), (K_{15}, E)\}$. Let $V = \{m, n, o, p\}$, $F = \{n_1, n_2, n_3\}$ and $Y = \{n\} \subseteq_{S_tM} V$ with $V/R = \{\{m\}, \{p\}, \{n, o\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F), (G_9, F)\}$ is a S_tN - Topology on V . Then $\delta = (G_{15}, F) \notin \sigma_{S_tN}(Y)$. Hence $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_9, F), (G_{15}, F)\}$, $S_tMC(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_2, F), (G_8, F)\}$. Let $W = \{a, b, c, d\}$, $L = \{l_1, l_2, l_3\}$ and $Z = \{a, b\} \subseteq_{S_tM} W$ with $W/R = \{\{a\}, \{b\}, \{c, d\}\}$. Then $\lambda_{S_tM}(Z) = \{\phi, (W, L), (H_6, L)\}$. Let $\delta = (H_3, L) \notin \lambda_{S_tN}(Z)$ and $S_tMO(W, \lambda_{S_tM}(Z), L) = \{\phi, (W, L), (H_3, L), (H_6, L)\}$. Let $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is a contra soft micro continuous and the mapping defined as $f(i) = m, f(j) = n, f(k) = o, f(l) = p$ and $\check{p}_1: E \rightarrow F$ by $\check{p}_1(m_1) = n_1, \check{p}_1(m_2) = n_2, \check{p}_1(m_3) = n_3$. Also, define $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is a contra soft micro continuous, then the mapping defined as $g(m) = b, g(n) = c, g(o) = d, g(p) = a$ and $\check{p}_2: F \rightarrow L$ by $\check{p}_2(n_1) = l_1, \check{p}_2(n_2) = l_2, \check{p}_2(n_3) = l_3$. Now, $gof: (U, \tau_{S_tM}(X), E) \rightarrow (W, \lambda_{S_tM}(Z), L)$ be the composition of two contra soft micro continuous function. Then gof is not contra soft micro continuous. Since $(gof)^{-1}(H_3, L) = f^{-1}(g^{-1}(H_3, L)) = f^{-1}(G_2, F) = (K_2, E)$ is not soft micro closed in $(U, \tau_{S_tM}(X), E)$.

Definition 4.10: Let $(U, \tau_{S_tM}(X), E)$ and $(V, \sigma_{S_tM}(Y), F)$ be a two S_tMTS . Then $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is said to be

- Contra Soft Micro Semi Continuous, (CS_tMS –Continuous)** if the inverse image of every soft micro-open set in $(V, \sigma_{S_tM}(Y), F)$ is S_tMS –closed in $(U, \tau_{S_tM}(X), E)$.
- Contra Soft Micro Pre Continuous, (CS_tMP –Continuous)** if the inverse image of every soft micro-open set in $(V, \sigma_{S_tM}(Y), F)$ is S_tMP –closed in $(U, \tau_{S_tM}(X), E)$.
- Contra Soft Micro β –Continuous, ($CS_tM\beta$ –Continuous)** if the inverse image of every soft micro-open set in $(V, \sigma_{S_tM}(Y), F)$ is $S_tM\beta$ –closed in $(U, \tau_{S_tM}(X), E)$.

5. CONTRA SOFT MICRO IRRESOLUTE FUNCTIONS

Definition 5.1: Let $(U, \tau_{S_tM}(X), E)$ and $(V, \sigma_{S_tM}(Y), F)$ be a two S_t MTS. Then $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is said to be

- (i) CS_tMS –irresolute, if $f^{-1}(G, F) \in S_tMSC(U, \tau_{S_tM}(X), E)$ for every $(G, F) \in S_tMSO(V, \sigma_{S_tM}(Y), F)$.
- (ii) CS_tMP –irresolute, if $f^{-1}(G, F) \in S_tMPC(U, \tau_{S_tM}(X), E)$ for every $(G, F) \in S_tMPO(V, \sigma_{S_tM}(Y), F)$.
- (iii) $CS_tM\beta$ –irresolute, if $f^{-1}(G, F) \in S_tM\beta C(U, \tau_{S_tM}(X), E)$ for every $(G, F) \in S_tM\beta O(V, \sigma_{S_tM}(Y), F)$.

Example 5.2: (i) Let $U = \{i, j, k, l\}, E = \{m_1, m_2, m_3\}$ and $X = \{j, l\} \subseteq_{S_tM} U$ with $U/R = \{\{i\}, \{j\}, \{k, l\}\}$. Let $\tau_{S_tN}(X) = \{\phi, (U, E), (K_3, E), (K_{11}, E), (K_{15}, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_2, E) \notin \tau_{S_tN}(X)$ and $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_2, E), (K_3, E), (K_6, E), (K_{11}, E), (K_{14}, E), (K_{15}, E)\}$. Let $V = \{m, n, o, p\}, F = \{n_1, n_2, n_3\}$ and $Y = \{n, p\} \subseteq_{S_tM} V$ with $V/R = \{\{n\}, \{m, o, p\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F), (G_3, F), (G_{14}, F)\}$ is a S_tN - Topology on V . Then $\delta = (G_{11}, F) \notin \sigma_{S_tN}(Y)$ and $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_3, F), (G_{11}, F), (G_{14}, F), (G_{15}, F)\}$. Define a function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ by $f(i) = m, f(j) = n, f(k) = o, f(l) = p$ and $\check{p}: E \rightarrow F$ by $\check{p}(m_1) = n_1, \check{p}(m_2) = n_2, \check{p}(m_3) = n_3$. Here, $f^{-1}(G_3, F) = (K_3, E)$ and $f^{-1}(G_{11}, F) = (K_{11}, E)$ and $f^{-1}(G_{14}, F) = (K_{14}, E)$ and $f^{-1}(G_{15}, F) = (K_{15}, E)$ are soft micro semi closed in $(U, \tau_{S_tM}(X), E)$. Therefore, f is CS_tMS –irresolute. Then $f^{-1}(G, F) \in S_tMSC(U, \tau_{S_tM}(X), E)$ for every $(G, F) \in S_tMSO(V, \sigma_{S_tM}(Y), F)$. Therefore, f is CS_tMS –irresolute and CS_tMP –irresolute.

(ii) Let $U = \{i, j, k\}, E = \{m_1, m_2, m_3\}$ and $X = \{k\} \subseteq_{S_tM} U$ with $U/R = i, j, k$. Let $\tau_{S_tN}(X) = \{\phi, (U, E), (K_4, E)\}$ is a S_tN - Topology on U . Then $\delta = (K_3, E) \notin \tau_{S_tN}(X)$. $S_tMO(U, \tau_{S_tM}(X), E) = \{\phi, (U, E), (K_3, E), (K_4, E), (K_7, E)\}$. Let $V = \{m, n, o\}, F = \{n_1, n_2, n_3\}$ and $Y = \{m, n, o\} \subseteq_{S_tM} V$ with $V/R = \{\{m\}, \{n, o\}\}$. Let $\sigma_{S_tN}(Y) = \{\phi, (V, F)\}$ is a S_tN – Topology on V . Then $\delta = (G_4, F) \notin \sigma_{S_tN}(Y)$ and $S_tMO(V, \sigma_{S_tM}(Y), F) = \{\phi, (V, F), (G_4, F)\}$. Define $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be a function defined as $f(i) = o, f(j) = m, f(k) = n$ and $\check{p}: E \rightarrow F$ by $\check{p}(m_1) = n_1, \check{p}(m_2) = n_2, \check{p}(m_3) = n_3$. Here, $f^{-1}(G_4, F) = (K_2, E)$, $f^{-1}(G_6, F) = (K_5, E)$ and $f^{-1}(G_7, F) = (K_6, E)$ are soft micro β –closed in $(U, \tau_{S_tM}(X), E)$. Hence f is $CS_tM\beta$ –irresolute.

Theorem 5.3: A function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is CS_tMS –irresolute (CS_tMP –irresolute, $CS_tM\beta$ –irresolute) iff the inverse image of every S_tMS –closed (S_tMP -closed, $S_tM\beta$ -closed) set in $(V, \sigma_{S_tM}(Y), F)$ is S_tMS –open (S_tMP -open, $S_tM\beta$ -open) in $(U, \tau_{S_tM}(X), E)$.

Proof: Let $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ be CS_tMS –irresolute. Let (G, F) be a soft micro semi closed set in $(V, \sigma_{S_tM}(Y), F)$. Then $(G, F)^c$ is soft micro semi open in $(V, \sigma_{S_tM}(Y), F)$. Since f is CS_tMS –irresolute, $f^{-1}((G, F)^c)$ is soft micro semi closed in $(U, \tau_{S_tM}(X), E)$. Therefore, $f^{-1}(G, F)$ is soft micro semi open in $(U, \tau_{S_tM}(X), E)$. Thus, the inverse image of every S_tMS –closed set in $(V, \sigma_{S_tM}(Y), F)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$.

Conversely, if the inverse image of every S_tMS –closed set in $(V, \sigma_{S_tM}(Y), F)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Let (J, F) be a S_tMS –open set in $(V, \sigma_{S_tM}(Y), F)$. Then $(J, F)^c$ is S_tMS –closed in $(V, \sigma_{S_tM}(Y), F)$. By our assumption $f^{-1}((J, F)^c) = (f^{-1}(J, F))^c$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Thus, the inverse image of every S_tMS –open set in $(V, \sigma_{S_tM}(Y), F)$ is S_tMS –closed in $(U, \tau_{S_tM}(X), E)$. Hence f is CS_tMS –irresolute.

Theorem 5.4: Consider $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ and $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ are CS_tMS –irresolute (CS_tMP –irresolute, $CS_tM\beta$ –irresolute), then gof is CS_tMS –irresolute (CS_tMP –irresolute, $CS_tM\beta$ –irresolute).

Proof: Let (H, L) be S_tMS –open set in $(W, \lambda_{S_tM}(Z), L)$. Since g is CS_tMS –irresolute, $g^{-1}(H, L)$ is S_tMS –closed in $(V, \sigma_{S_tM}(Y), F)$. Since f is CS_tMS –irresolute, $f^{-1}(g^{-1}(H, L)) = (gof)^{-1}(H, L)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Thus, the inverse image of every S_tMS –open set in $(W, \lambda_{S_tM}(Z), L)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Therefore, gof is CS_tMS –irresolute.

Theorem 5.5: If the function $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ is CS_tMS –irresolute (CS_tMP –irresolute, $CS_tM\beta$ –irresolute) and the function $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ is CS_tMS –Continuous (CS_tMP - Continuous, $CS_tM\beta$ -Continuous) then gof is S_tMS –Continuous (S_tMP - Continuous, $S_tM\beta$ – Continuous).

Proof: Let (H, L) be S_tM –open set in $(W, \lambda_{S_tM}(Z), L)$. Since g is CS_tMS –Continuous, $g^{-1}(H, L)$ is S_tMS - closed in $(V, \sigma_{S_tM}(Y), F)$. Since f is CS_tMS –irresolute, $f^{-1}(g^{-1}(H, L)) = (gof)^{-1}(H, L)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Thus, the inverse image of every S_tM –open set in $(W, \lambda_{S_tM}(Z), L)$ is S_tMS –open in $(U, \tau_{S_tM}(X), E)$. Therefore, gof is S_tMS –Continuous.

Theorem 5.6: Consider $f: (U, \tau_{S_tM}(X), E) \rightarrow (V, \sigma_{S_tM}(Y), F)$ and $g: (V, \sigma_{S_tM}(Y), F) \rightarrow (W, \lambda_{S_tM}(Z), L)$ be any two soft micro function.

(i) If f is CS_tMS –irresolute and g is S_tMS –continuous then gof is CS_tMS –continuous.

(ii) If f is S_tMS –irresolute and g is CS_tMS –irresolute then gof is CS_tMS –irresolute.

Proof: The proof is obvious.

6. CONCLUSION

In this paper, we introduced the concept of **Soft Micro Irresolute**, **Contra Soft Micro Continuity** and **Contra Soft Micro Irresolute** and studied its basic properties. We also defined some other Irresolute and Contra Continuity and Contra Irresolute functions in Soft Micro Topological Spaces. The obtained results provide a basis for further research in soft micro topological spaces and their applications.

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